

Definitions:

The force of attraction between two molecules of the same substance is called **cohesive force**. Property is called **cohesion**.

The force of attraction between two molecules of different substances is called **adhesive force**.

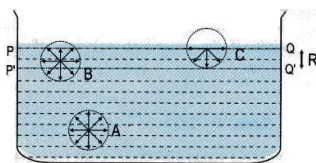
The maximum distance between two molecules up to which intermolecular forces are effective is called **range of molecular attraction**.
NOTE: Intermolecular forces are effective in solids & liquids for 10^{-9}m only

An imaginary sphere drawn, with given molecule as the centre and radius equal to the molecular range is called **sphere of influence**.
NOTE: In sphere of influence every molecule inside will attract the molecule at the centre and this central molecule will also attract those with its sphere of influence.

Surface tension on the basis of Molecular Theory :

Consider three molecules A, B and C of the liquid along with their sphere of influence

- Molecule A is well within the liquid and its sphere of influence is completely within the liquid. It is acted upon by equal and opposite cohesive forces. Therefore the net force of attraction on A is zero.
- Molecule B is just inside the surface such partly inside the liquid and partly outside. The number of air molecules and its attraction is negligible as compared to the liquid molecules. Thus, B is acted upon by a net downward force.
- Molecule C is on the liquid surface such that half its sphere of influence is inside and half outside. Thus it is acted upon by a maximum downward force.
- Whenever molecules from the inside of a liquid come up to the surface, work is done against the downward force. This work done is stored as PE within the surface film. Thus, more the molecules on the surface, more will be its PE. But a body always tries to attain minimum PE. Thus it tries to contract (molecules are pulled inside the liquid due to the net downward force), in order to keep minimum number of molecules on the surface. This gives rise to surface tension which acts tangentially to the liquid surface.



Define Surface Tension

The force per unit length acting at right angles to an imaginary line drawn on the free surface of liquid is called **surface tension**.

$$T = \frac{F}{l}$$

S.I. Unit : N/m or J/m², CGS Unit : dyne/cm,
 Dimensions: [M¹ L⁰ T⁻²]

Some applications

- Tooth paste spreads more freely in the mouth while cleaning the mouth because it contains soap which reduces surface tension
- If detergent is added in water, the surface tension reduces. This increases area of contact and cleaning ability of soap solution.
- Mosquito eggs float on the water surface. Spraying kerosene on the water surface lowers the surface tension and the eggs go inside the water and the breeding of mosquitoes stops.

Relation between surface tension and surface energy:

Consider a rectangular frame ABCD, on which the wire CD can slide without friction. This frame is dipped inside soap solution, taken out and held horizontally.

'L' : length of wire CD

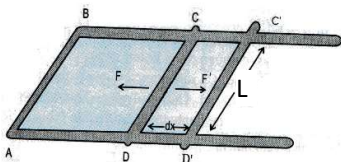
T : surface tension of the soap solution
 The wire CD experiences an inward force $F = 2 T L$ (as there are two sides to this film). To increase the surface area of the liquid, and external force F' equal and opposite to F is applied to CD under isothermal conditions. CD moves to C'D' through distance dx .

$$\text{Work done against } F = dw = F' dx = 2 T L dx = T dA$$

where $dA = 2 L dx$ = increase in the area of the two surfaces

This work done is stored in the film in the form of potential energy which is called **surface energy**.

Surface Energy = $T dA$.



Thus surface tension is numerically equal to surface energy per unit area.

$$T = \frac{dW}{dA}$$

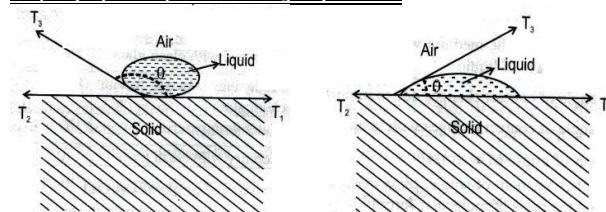
Angle of contact

When a liquid is in contact with a solid, the angle between the tangent drawn to the free surface of the liquid and the surface of the solid at the point of contact measured inside the liquid is called **angle of contact**.

Characteristics :

- >> For a given solid – liquid pair, the angle of contact is constant
- >> angle of contact depends upon the nature of liquid and solid in contact
- >> It depends upon medium which exists above the free surface of liquid
- >> angle of contact changes due to impurity
- >> angle of contact changes with temperature

Shape of liquid drop based on angle of contact:



T_1 : Force of surface tension at liquid – solid interface

T_2 : Force of surface tension at air – solid interface

T_3 : Force of surface tension at air – liquid interface

For equilibrium of the drop

$$T_2 = T_1 + T_3 \cos \theta \quad \text{i.e.} \quad \cos \theta = \frac{T_2 - T_1}{T_3}$$

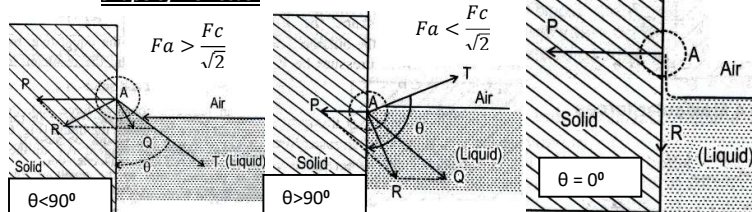
>> $\cos \theta$ is positive, θ is acute when $T_2 > T_1$ and $T_2 - T_1 < T_3$

>> $\cos \theta$ is negative, θ is obtuse when $T_2 < T_1$ and $T_1 - T_2 < T_3$

>> $\cos \theta$ is 1, θ is zero when $T_2 = T_1 = T_3$

>> $\cos \theta > 1$, not possible when $T_2 - T_1 > T_3$ (liquid will spread over the solid surface and drop would not be formed).

Shape of meniscus:



Case 1: In case of liquid (eg kerosene) partly wets the solid, resultant adhesive force (AP) between liquid and solid acting on molecule A is stronger than resultant cohesive force (AQ) between liquid molecules. Therefore resultant force of AP and AQ, AR lies inside the solid. In equilibrium state, tangent AT to the liquid surface, must be perpendicular to the resultant force. Hence the liquid surface in contact with the solid is concave upwards and the angle of contact is acute.

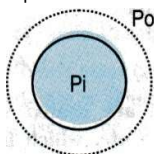
Case 2: In case of liquid (eg mercury) which does not wet the solid, resultant adhesive force (AP) between liquid and solid acting on molecule A is smaller than resultant cohesive force (AQ) between liquid molecules. Therefore resultant force of AP and AQ, AR lies inside the liquid. In equilibrium state, tangent AT to the liquid surface, must be perpendicular to the resultant force. Hence the liquid surface in contact with the solid is convex upwards and the angle of contact is obtuse.

Case 3: In case of liquid (eg pure water) which completely wets the solid, resultant adhesive force (AP) between liquid and solid acting on molecule A is very strong than resultant cohesive force (AQ) between liquid molecules. Therefore resultant force of AP and AQ, AR lies along AP. In equilibrium state, tangent AT to the liquid surface, must be perpendicular to the resultant force. Hence the liquid surface in contact with the solid is vertical upwards and the angle of contact is zero.

EXCESS PRESSURE IN DROPS AND BUBBLES

Neglecting the effects of gravity and air resistance, the shape of drops and bubbles are spherical due to surface tension.

Drops: As it is spherical, the inside pressure is more than the outside pressure. P_o : outside pressure, P_i : inside pressure, $P_i - P_o$: excess pressure. Let radius of the drop increase from r to $r + \Delta r$, so that the inside pressure remains almost constant.



$$\text{Initial surface area} = A_1 = 4\pi r^2$$

$$\begin{aligned}\text{Final surface area} &= A_2 = 4\pi(r + \Delta r)^2 = 4\pi(r^2 + 2r\Delta r + \Delta r^2) \\ &= 4\pi r^2 + 8\pi r\Delta r + 4\pi\Delta r^2 = 4\pi r^2 + 8\pi r\Delta r \\ &\quad (\text{Since } \Delta r \text{ is very small, } \Delta r^2 \text{ term is neglected})\end{aligned}$$

$$\text{Increase in surface area} = dA = A_2 - A_1 = 4\pi r^2 + 8\pi r\Delta r - 4\pi r^2 = 8\pi r\Delta r$$

$$\text{Work done to increase this surface area} = dW = T dA = T (8\pi r\Delta r) \dots (i)$$

$$\text{But work done} = dF \times \Delta r \quad \text{and} \quad dF = dP \times A = (P_i - P_o) 4\pi r^2$$

where dP : excess pressure and dF : excess force

$$\text{Thus } dW = dF \Delta r = (P_i - P_o) 4\pi r^2 \Delta r \dots (ii)$$

$$\text{From (i) and (ii), } T (8\pi r\Delta r) = (P_i - P_o) 4\pi r^2 \Delta r$$

$$\text{Hence, } P_i - P_o = \frac{2T}{r} \quad \text{This is called Laplace's law of spherical membrane}$$

Bubbles: They have two surfaces in contact with air. Hence,

$$\text{total increase in surface area} = dA = 2(8\pi r\Delta r) = 16\pi r\Delta r$$

$$dW = T dA = 16\pi r\Delta r T \dots (i)$$

$$\text{Work done due to excess pressure} = (P_i - P_o) \times 4\pi r^2 \Delta r \dots (ii)$$

$$\text{From (i) and (ii), } (P_i - P_o) \times 4\pi r^2 \Delta r = 16\pi r\Delta r T$$

$$\text{Hence, } P_i - P_o = \frac{4T}{r}$$

CAPILLARITY

The phenomenon of rise or fall of a liquid inside a capillary tube when it is dipped in the liquid is called **capillarity**.

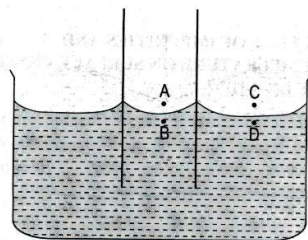
Applications:

- Sap & Water rises up to the topmost leaves in the tree
- Water rises up the crevices in rocks
- Cloth rag soaks water
- Ink rise in a pen
- Blotting paper absorbing ink
- Oil rise up the wick of lamp

Explanation:

Suppose a capillary tube is dipped into liquid like water which wets the capillary. Let us consider a situation before the movement takes place inside the capillary.

- A : above the concave surface inside the capillary
- B : below the concave surface inside the capillary
- C : above the plane surface outside the capillary
- D : below the plane surface outside the capillary



Pressure on the concave side is more than the convex side.

$$\text{Thus, } P_A > P_B$$

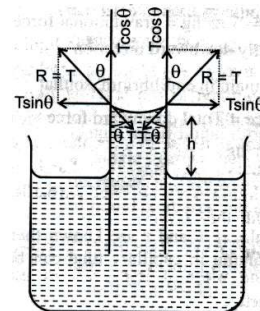
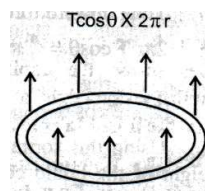
$$\text{But, } P_A = P_C \text{ (atmospheric pressure)}$$

$$P_C = P_D, \text{ pressure is same on both sides of a plane surface}$$

$$\text{Thus } P_A = P_C = P_D$$

But B and D are at the same horizontal level inside the liquid, hence their pressure should be same. But from above it's clear that $P_D > P_B$. Therefore liquid is not in equilibrium. Liquid flows from high pressure D, towards low pressure B. This liquid rises in the capillary tube till pressure at B becomes same as that at D.

For liquid which does not wet solid (like mercury), meniscus of liquid inside the capillary is convex and the fall of mercury level in a glass capillary can be explained in similar way.

Rise in height of liquid in capillary tube

Consider a glass capillary of a small radius r , dipped into a liquid, which wets the glass. Liquid rises in the capillary to a height h . Let T be the surface tension of the liquid. Thus a force of T acts per unit length of liquid surface in contact with the walls of the capillary. This force is along the tangent to the liquid concave meniscus making an angle θ with the wall of capillary and acts inwards. The liquid meniscus is along a circle of circumference $2\pi r$. By Newton's third law, an equal and opposite reaction R acts directed outwards.

The force of reaction T acting per unit length can be resolved into

>> vertical component $T\cos\theta$ in the upward direction

>> horizontal component $T\sin\theta$ acting radially outward.

All horizontal components cancel each other, while all vertical components act along the circumference of meniscus, add to each other. This rises the liquid in the capillary tube.

$$\text{Total vertical upward force} = T\cos\theta (2\pi r)$$

This total upward force balances the weight of the liquid column in the capillary tube.

$$W = Mg = V\rho g = \pi r^2 h \rho g \quad \text{where } \rho: \text{density of liquid}$$

$$M: \text{mass of liquid column inside the capillary}$$

Thus for equilibrium,

$$2\pi r T \cos\theta = \pi r^2 h \rho g$$

$$\text{Thus } T = \frac{r \rho g}{2 \cos\theta}$$

Thus, if T and r are known, height of the liquid column can be found as

$$h = \frac{2T \cos\theta}{r \rho g}$$

Thus, rise in the liquid column is inversely proportional to the radius of the capillary tube and density of liquid

NOTE: For water $\theta = 0^\circ$, $\cos 0 = 1$, $\rho = 1 \text{ g/cc}$, Thus, $T = \frac{r \rho g}{2} \text{ dyne/cm}$

NOTE: We measure the height of liquid at meniscus horizontally. But the liquid is above this level. The actual equation, the height of liquid should be taken as $h + r/3$ but since $h \gg r$, $r/3$ is neglected.

NOTE: In absence of gravity, $h = \text{infinite}$. Thus liquid will rise to the top of the tube.

NOTE: on moon, where the gravity is $\frac{1}{6}$ th that of the earth, the height of the capillary will be six times that at the surface of the earth.

FACTORS AFFECTING SURFACE TENSION**Impurities:**

>>For **highly soluble** impurities like common salt in water, surface tension **increases**.

>>For **sparingly soluble** substances like phenol or alcohol in water, surface tension **decreases**

>>For **insoluble** impurity added in water, surface tension **decreases**

eg. We clean dirty cloths containing oil stains by adding detergents (or soap) to water. It reduces the surface tension, water now can easily penetrate through the pores of the clothes and can clean the dirt on the clothes.

eg. By sprinkling oil in the sea, water calms down, due to decrease in surface tension of water. And thereby the height of water is also reduced.

Temperature: In most liquids, increase in temperature causes **decrease** in surface tension (exception copper, molten cadmium). The temperature at which surface tension of liquid becomes zero is called **critical temperature** of the liquid.

If T is the surface tension of liquid at $\theta^\circ\text{C}$ and T_0 at 0°C , then over a certain range of temperature $T = T_0(1 - \alpha\theta)$, where α is a constant which depends on the nature of the liquid

Contamination: Presence of dust particles or lubricants on the liquid surface **decreases** its surface tension.

